Numbers

By Albert Botha

Despite appearances to the contrary, the formulas on the right are considered to be stock standard option pricing formulas. In today's world of finance they are very important, but they are by no means the most complicated.

$$c = P(0,T)[F_0\Phi(d_1) - X\Phi(d_2)]$$

$$\Phi(x) \sim N(0,1)$$

These pricing formulas and most of the Greek letters used in them can be further expanded to produce monstrous mathematical creations beyond the understanding (or interest) of all but a handful of people.

$$N(\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-u)^2}{2\sigma^2}\right)$$

Despite the complexity, the field of financial mathematics and the underlying science of statistics are still very young, especially when compared to the classical sciences of mathematics, physics or even biology.

$$d_1 = \left[\frac{\ln(F_0/X) + (\sigma^2 T/2)}{\sigma\sqrt{T}}\right]$$

To understand how financial mathematics is used and what its shortcomings are, one needs to understand something of its history, its youth and the premises it is based on.

$$d_2 = \left[\frac{\ln(F_0/X) - (\sigma^2 T/2)}{\sigma\sqrt{T}}\right]$$

Some scholars put the birth date of statistics at 1662. This is the date that John Graunt published a book on the analysis of human mortality in London. In this book he used statistics to estimate the population size of London as well as producing the world's first life table, which can be used to estimate probability of survival to each age. The important words here are "estimate" and "probability". It is not an exact science.

Also during the 17th Century Blaise Pascal and Pierre de Fermat built the mathematical foundations on which statistics was founded. They did this by studying games of chance.

Since its birth, many great minds made contributions to the field, including luminaries like Albert Einstein who used Brownian motion to indirectly confirm the existence of atoms. However it was the advent of the computer, which allowed people to perform large-scale statistical calculations, which truly allowed statistics to be used (and abused) to its fullest potential.

From the field of statistics grew the field of financial mathematics. It is also called financial engineering or financial risk management, yet the end result remains the same – It is using maths to allow individuals, corporations and funds to manage risk in the "correct" way.

By selecting your assets and liabilities and structuring your investment in a certain way, you optimise your exposure to risk – this in turn allows you to take on more risk as you are more certain as to the possible range of outcomes.

Here it is important to remember that financial mathematics is a focused field of statistics and the statistics deal in probabilities, not in certainties. It analyses past data to try and estimate the risk inherent in certain combinations of investment. This means that your predictions are only as

complete as your data and the calculations you can make from them. By looking at the past we can model our behavior to avoid risk similar to those in our data set.

There is however a very important issue here that people miss. Risk and uncertainty do not come from knowable, quantifiable events – they spring from improbability and unpredictable events. It is exactly those events we did not predict which cause problems. It is the problem of the "Black Swan"[1].

Large financial corporations use a measure called Value at Risk (VaR) to financially quantify how much of their capital they could potentially lose over the next day, week or month. When financial decisions are made, the effect on the company's bottom line is viewed in light of the potential effect on the VaR. The calculations used by the companies are usually fine-tuned so that they give reasonable results in 99.5% of circumstances. The problem is, as recently, sometimes a 0.05% event does occur, a perfect storm, for which the model is not calibrated. An event it could not predict and that it cannot protect you against.

This is where the problem comes in. Fund managers, CEOs and corporations became overly reliant on these models. They expected them to be perfect. They thought they offered complete protection against uncertainty. This allowed them to take on massive leverage and huge amounts of risk, because the model told them that in 99.5% of circumstances they would be protected. So when the perfect storm struck, the risk and leverage combined to do what they do best and caused mass financial devastation.

This amalgamation of effects has given rise to a distrust in models and even the word "quant"[2] is somewhat reviled. The problem however was never with the models. Models by their very nature are only an approximation of reality and are limited by their input data. They cannot be perfect predictors of the future and they should not be regarded as such. However they are, and should remain, an integral part in the decision-making process.

These days there is just too much information and the systems are too complex for a single person to assimilate. Quant models help process and simplify the information. It is however important that this information be interpreted. Quant models are not the Holy Grail, and their outputs are inviolate. It is when people forget this fact that problems start.

Quant models and their application in the modern financial industry will continue to improve. Going forward however it will serve investors and corporations alike to remember that the financial game has a substantial human aspect to it. And that as such, no model will ever be able to completely describe all its intricacies. All models and simulations, no matter how subtle, need a human hand and mind from time to time, both for upkeep and interpretation. Decisions should never be based completely on the output from a "black box".

"Many of today's policy proposals start from the view that "greed" and "incompetence" and "poor risk assessment" are the ultimate source of what went wrong. In fact, they were not the true cause at all. Moreover, even if they had been, it is fatuous to think that we will now create a post-crash generation of bankers and traders who are not greedy, much less a new generation of quants who will be able to assess and manage risks much better than "the idiots" who have brought us to the current abyss. Greed cannot be exorcised. Nor can the inherent inability of any quants to determine the "true" probability distributions of all-important events whose true probabilities of occurrence can never be assessed in the first place."

1 – "Fooled by Randomness" and "Black Swan", both written by Nassim Taleb deal with the impact of highly improbable/unpredictable events. Definitively a read I would recommend.

2 – A "quant" is the short industry term for a quantitative analyst, a person who spends a lot of time doing calculations and building quantitative (quant) models.